UofSC, Department of Physics & Astronomy.

Graduate student problem competition

January 23–30, 2023

All graduate students are eligible to participate.
To submit your solution, e-mail it to `bazaliy@mailbox.sc.edu`

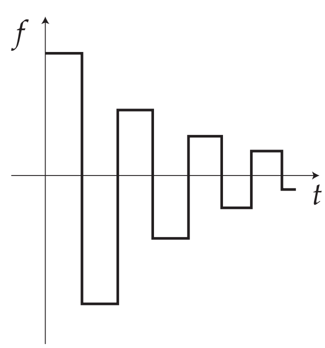
Excitation by square wave force

Quantum harmonic oscillator in one dimension is described by a Hamiltonian

$$\hat{H}_0 = \frac{\hat{p}^2 + \hat{x}^2}{2}$$

(we set $\hbar = 1$, so $\hat{p} = -i\, d/dx$). It is prepared in its ground state at $t < 0$. At $t = 0$ an external force $f(t)$ starts acting on it, so that Hamiltonian acquires a form

$$\hat{H}(t) = \hat{H}_0 - f(t)\hat{x} \quad (t \geq 0)\ .$$



Applied force has a decaying square wave shape (see Figure), described by

$$\begin{array}{rcll} f(t) & = & +f_0 & (0 < t < T/2), \\ f(t) & = & -f_0 & (T/2 < t < T), \\ f(t) & = & +f_1 & (T < t < 3T/2), \\ f(t) & = & -f_1 & (3T/2 < t < 2T), \\ & \vdots & & \end{array}$$

$$
\begin{aligned}
f(t) &= +f_n \quad (nT < t < nT + T/2), \\
f(t) &= -f_n \quad (nT + T/2 < t < (n+1)T), \\
&\vdots
\end{aligned}
$$

Decaying amplitudes are given by

$$f_n = \frac{f_0}{2^n} \ .$$

Parameter $T = 2\pi$ is set so that one cycle of the square wave is equal to the period of the unperturbed oscillator.

As we know, periodic time dependent perturbations excite the system from ground state to higher energy states. Usually, textbooks discuss the influence of harmonic exciting potentials. Your task will be to study a square wave instead. Find the leading order approximation (in $f_0 \to 0$) for the probability to find the oscillator in its first excited state at $t \to \infty$.